# Transient Analysis

## Intro

### ILOs

Upon completion of this week’s readings, you should be able to demonstrate knowledge in the following:

1. Analyze first order transient circuits in switching events:

2. Find initial and final steady state values

3. Determine the time constant of an RL or RC circuit

4. Express the current or voltage as a function of time.

**Diagram, schematic

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### Topic 3 Videos:

1. Analytical & Multisim: <https://www.youtube.com/watch?v=IzEs7ySSSSk&list=PLhbHWgMknRJT_eKLFXB843NkaNHfJ37Pw&index=12>
2. Lab Skills: <https://www.youtube.com/watch?v=-sFz0Wwc4Zs&list=PLhbHWgMknRJT_eKLFXB843NkaNHfJ37Pw&index=13>
3. Extra Practice: <https://www.youtube.com/watch?v=w0tLpEkownQ&list=PLhbHWgMknRJT_eKLFXB843NkaNHfJ37Pw&index=14>
4. Hantek Lab Modification: <https://youtu.be/Nojw_gDxgiM?list=PLhbHWgMknRJT_eKLFXB843NkaNHfJ37Pw>

### Deliverables

Note: As always, your full objective for this topic is to review these notes, the videos, practice problems, live class sessions and forum content, then to write-up creating and tri-solving a variation problem of the week's topic that demonstrates you've mastered the content. With that all still in mind, following are some specific guidelines & tips for this week.

See the Outline and Deliverable Rubric files for information for the write-ups in general. Specific tasks for your H3 deliverable are as follows:

1. Build a circuit similar to the sample lab (i.e., using only resistors, capacitors, a switch, and a DC voltage supply arranged so that the capacitor(s) will charge in one switch configuration and discharge in the other.)
   1. **Note: make sure that whether charging or discharging you have resistors in the way of the current so you have a nonzero timeconstant; i.e., you're not trying to instantly charge or discharge a capacitor by shorting it out or putting it in parallel with your supply. Doing so will damage your equipment.** (right after a switching event the capacitor acts like a perfect supply able to push any current to maintain its voltage, so will short out components if there isn't a resistor in the way of that current to limit it)
   2. Note: only use the ceramic capacitors; don't use the electrolytic capacitor (because you don't need it and it can be dangerous if used incorrectly).
2. Analytical:
   1. Write an analytical expression for the voltage across the capacitor as a function of time when the capacitor begins fully discharged at *t* = 0 and the is suddenly toggled to the position where it will charge. What is the RC time constant of the circuit in this configuration? Show all your work.
   2. Repeat this process for a discharge switching event.
3. Use multisim to confirm the charging and discharging switching time constants for your circuit. Show scope measurements that capture these charge and discharge events on appropriate scales to measure the time constant, and [briefly] explain your process.
4. Physically build your circuit on the breadboard and use the scope to measure the time constants by capturing charging and discharging events.
   1. Hint: You likely want to create your "switch" but plugging and unplugging a wire into the breadboard rather than the "switches" in the component kit, because those switches are only active when pressed (i.e., they don't toggle).
5. For the physical build only: Exercise to measure the scope probe impedance
   1. Connect the oscilloscope across the capacitor when it’s fully charged and then disconnect the capacitor from the circuit (while leaving it connected to the scope only). What is the initial voltage across the capacitor? How much energy is stored in the capacitor at this voltage? What is discharging the capacitor? Estimate the input impedance of the oscilloscope probes in this way.
   2. *Note: this doesn't work with multisim, because multisim treats the scope as ideal and gives it infinite impedance.*
   3. *May be helpful:* [*https://youtu.be/Nojw\_gDxgiM?list=PLhbHWgMknRJT\_eKLFXB843NkaNHfJ37Pw&t=415*](https://youtu.be/Nojw_gDxgiM?list=PLhbHWgMknRJT_eKLFXB843NkaNHfJ37Pw&t=415)
6. Analysis: discuss agreement or disagreement between your results and their meaning [as usual]; in particular, discuss whether the physical and multisim curves you observed are consistent with your analytical expressions.

## Transient Analysis

### Transient Analysis

Transient analysis means determining what occurs as the circuit moves from one steady state to another after a change (i.e., opening or closing a switch).

**First-order circuit:** A circuit with one storage element (capacitor or inductor), or multiple ones which can be reduced to one (i.e., two capacitors in parallel or series is equivalent to one from the perspective of other elements in a circuit).

**Second-order circuit:** Has more multiple storage elements (which can't be reduced to one).

### Elements of Transient Problem Solving

***1. You can find steady-state values (for DC sources) by treating capacitors as opens and inductors as shorts.***

→ This is because in steady-state DC, nothing is changing in time. If there *were* DC current through a capacitor it would be accumulating charge and increasing voltage, meaning it hasn't reached steady-state yet. Therefore, if a capacitor voltage is constant (i.e., in steady-state DC) the current through it must be zero → it's an open circuit.

→ Same for an inductor but with voltage; an inductor only has voltage across it if the current through it is changing in time. Steady-state DC current through it means no voltage across it → it's a short circuit.

***2. You can find initial values for capacitor voltage and inductor current after a switching event by noting that neither can change instantly****.*

→Capacitor voltage is proportional to charge on the plates, and charge on the plates can't change instantly. This is because charge being there requires current to have flowed over some time; , and if the time interval is zero (i.e., an instant switching event), only an infinite current could make the integral nonzero, but infinite current is not physical. Therefore, capacitor voltage can't change instantly (i.e., is continuous).

→ Same for an inductor but with current; since , an instantly changing current would require infinite voltage. You can see this as well in integral form; ; inductor current can't change without time if inductor voltage is finite (and it is because infinite voltage isn't physical).

**3*. A voltage or current in a first-order circuit follows* , *where*  *is its initial value,*  *is its final value, and*  *or*  *is its* time constant**

→ here you use the **equivalent** *R* seen by the inductor or capacitor, with the same method & logic as finding Thévenin equivalent resistance seen by as load..

### First-Order Transient Analysis

1. Find the initial and steady state values of variable(s) you're interested in
2. Reduce the circuit if necessary to find the equivalent R seen by the passive storage element (inductor or capacitor)
3. Write  for the variable(s) of interest.

Tips summary:

1. *Voltage* across a *capacitor* can't change instantly: it's the same before and after switching events.
2. *Current* through an *inductor* can't change instantly: it's the same before and after switching events.
3. Use these initial capacitor and inductor signal values after the switch to determine initial currents and voltages everywhere by treating capacitors as DC voltage supplies and inductors as DC current supplies (at least *initially*).
4. To find  or  for complicated circuits, combine elements to express as a single effective capacitor or inductor with a single effective resistor. When doing this, it's useful to find the Thevenin circuit seen by the inductor or capacitor to find the resistance they see.
5. In circuits with multiple switching events (i.e., switch #1 opens 5 s before switch #2 opens) you can use the equation from the first switching event () to determine the initial condition before the second switching for your values that don't change (capacitor voltage and inductor current).

#### Example (taken from a 2018 test):

The following circuit is initially in steady-state when the switch is closed at time t = 0.

Diagram

Description automatically generated

1. **Find the time constant before and after the switch is thrown.**
2. **Find the current going through the inductor for all time *t* > 0.**

Solutions:

1. Before the switch is thrown, we have L/R1 = 0.1s

After the switch is thrown, we have L/(R1||R2) = 1 H/5 Ohm = 0.2 s

 (downward)

After a long time the inductor is again a short circuit (), so  (downward)

therefore,



(or ,

or )

#### Proof of first order transient analysis

OK, sure, , but where does this stuff come from? The answer: DE solving, current-voltage relations, and Kirchoff's laws. Let's do an example!

In this circuit

Diagram, schematic

Description automatically generated

All elements are in series, so KVL gives , where the relations between voltage across and current through the passives are  and  (note that the rate of change of capacitor charge is the current, ).

So, differentiating KVL and substituting these relations in, we have:



This is a first-order autonomous linear homogeneous ODE with constant coefficients. You can solve it in several different ways, e.g., separation of variables:



Only the positive root allows this to make sense at time *t* = 0, so it's . Because the final

Then to find the voltage across the capacitor, use the state equation:



Suppose the capacitor is initially uncharged when this battery is connected; then , so , which is also the final voltage on the capacitor, and we have .

More generally, suppose the capacitor initial voltage is  when the battery is connected. The initial current is then  (using KVL), and so, again noticing that either way the final voltage is , and defining , we have:

,

Or, rearranging, 

This was also [more subtly] true for the current equation we found; for this circuit, the final current is , so it's also true to say that .

### Second-Order Transient Analysis

Here the simple solution form and idea of a time constant from first-order doesn't apply any more.

To analyze:

1. Solve for initial and final steady-state values, (x(0) and x(∞))
2. Use Kirchoff's laws and the i-v relations for the capacitors / inductors to write a 2nd-order ordinary differential equation (ODE) for the state variable of interest (current through the inductor or voltage on the capacitor), and
3. Solve the DE.

The DE here will have the form , and each term has physical significance:

 is the **natural [resonant] frequency** of the circuit

 is the dimensionless **damping ratio**

 is the **forcing function** (due to the source(s))

 is the **DC gain** of the variable *x*.

*Next week we look at these circuits in response to AC with phasor analysis and show how we can exploit their resonant characteristics to create "filters" (electronic devices that selectively reduce signals based on their frequency).*

The size of the damping ratio determines how the circuit will respond to a change event (switch open or close):

1. Overdamped (): signal slowly changes exponentially from initial to final value
2. Critically damped : signal changes as fast as possible towards equilibrium without ever passing that point
3. Underdamped (): signal crosses equilibrium point, oscillating at the damped natural frequency  (the amplitude still decays in time exponentially, but not as quickly as with critical damping)

## Tips on the Scope

The most challenging part of this topic is learning how scope triggering works so you can capture your rising and falling events. There are challenges in this in both multisim, and the physical circuit.

### Multisim tips

Zooming in the timescale for the scope in multisim may not be enough to get it to simulate the accuracy you're looking for, if multisim got the wrong idea about what details are important to you on what scale; check out this funky brain-teaser 1st-order circuit simulation:

Diagram, schematic

Description automatically generated

Now, on this timescale the switching event looks instant (and it should be), but if you zoom in the time scale after this single-sequence you can get it to spread out in time:

Graphical user interface

Description automatically generated

Now, if you re-take this sweep on this zoomed-in timescale, it doesn't actually get better!

The reason is that Multisim is only simulating a certain minimum auto-calculated timestep; you can change it though:

Graphical user interface, application

Description automatically generated

Now it will catch the switching event in less time:

Graphical user interface

Description automatically generated

Note that the auto-determined timestep was actually fine for finding the timeconstant of this circuit, as you can still see the exponential relationship at the start.

However, when I change the capacitor to 5pF the timeconstant is so short that you'll need to change the max timestep by quite a lot (down to around 1e-10) to see anything happening!

So, if your curve looks like this:

A screenshot of a computer

Description automatically generated with medium confidence

Then you have some big sampling problems that are throwing off your data; multisim is only actually measuring at the red dots here and then connecting them:

Graphical user interface, chart

Description automatically generated

So in reality, the start of the exponential rise after the switching event is not where the scope last took a datapoint at the bottom (the first red dot you were measuring from) but actually somewhere between that point and the next data point; far enough along that the slope would decrease all along the way after the switching event.

Even if you get a better looking curve, be very careful with short time constants that you don't fool yourself and mis-measure things. For example, this might look like a good rising event curve:

Graphical user interface

Description automatically generated

But it's actually still mismeasured and has a sampling problem - look closely at the slope at the start:

A picture containing diagram

Description automatically generated

the slope is increasing rather than starting at a max right after switching and decreasing as time moves on:

A picture containing chart

Description automatically generated

This means you should reduce the max timestep in multisim as shown above.

Note also that the left time cursor here also missed a bit; it's a bit to the right of the last low datapoint, so if this was your measure you might have cancelled out the sampling error by making a cursor measurement error. (Like when you make two mistakes on a multiple choice math test and end up with the right answer because they cancelled out. Lucky, but we can do better!)

### Physical tips

For one example of a challenge in the physical circuit, here's a student's data that could not seem to capture the charging events:  
A picture containing calendar

Description automatically generated  
…but discharging ones worked fine:

A screenshot of a video game

Description automatically generated with medium confidence

The problem is that the noise in the signal (especially during plugging in or unplugging a wire) were high enough for the scope to capture them at its trigger level (the yellow tab on the right)

A picture containing text, device

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(see how the centre spike goes above this?)

The solution is to raise the trigger even more when capturing a charging event:

A screenshot of a computer

Description automatically generated with medium confidence

(and possibly lower it down a bit again prior to trying to capture a discharge so you don't have the same problem from noise there:

A picture containing text, electronics

Description automatically generated

Also, toggling the switch with confidence can help reduce the noise during this event)

Be sure to check out the lab videos for information on how scope triggering works in detail:

1. Lab Skills: <https://www.youtube.com/watch?v=-sFz0Wwc4Zs&list=PLhbHWgMknRJT_eKLFXB843NkaNHfJ37Pw&index=13>
2. Hantek Lab Modification: <https://youtu.be/Nojw_gDxgiM?list=PLhbHWgMknRJT_eKLFXB843NkaNHfJ37Pw>